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RESEARCH ARTICLE

Image Compression based on Non-Linear Polynomial Prediction Model

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Abstract
This paper introduced a new image compression techniques based on using the modelling concept of polynomial second order Taylor series representation of nonlinear base. The results showed highly performance in terms of compression ratio and quality even with more complexity of coefficients estimation.

1. Introduction
Compression is the hart of multimedia subject, that control the amount of information saved and/or sends, implicitly affected computers and/or communication to reduce storage costs and/or transfer rate respectively.
A compression system generally, achieved by reducing the size of information required by exploited the redundancy. Image compression based on utilizing the redundancy within the image itself (i.e., image’s structure or features) – referred to as statistical redundancy, along with the utilization of the limitations of human vision or perception – referred to as psycho-visual redundancy. Hence, lossless and lossy techniques available. In other words, lossless also called information preserving or error free techniques based on exploiting the statistical redundancy alone, in which the image compressed without losing information with low compression performance, basically based on rearrange or reorder the image content. On the other hand, lossy based on exploiting the psycho-visual redundancy, either solely or combined with statistical redundancy, in which remove content from the image, that degrades the compressed image quality with high compression performance. Reviews of lossless and lossy techniques can be found in [1-8].
The prediction coding techniques based on modelling concept increasingly used in image compression, due to simplicity, symmetry of encoder/decoder and flexibility of use are the most significant advantages of this technique [9]. The core of prediction coding lies in the design of mathematical models, in which predicting or estimating each pixel value from nearby or neighbouring pixels, and then followed by finding the differences between the predicted value and the actual value that called residual or prediction error [10-12].
Today, the polynomial linear based adopted by [13], and followed by [14-18] to compress the images effectively based on modelling distance between image pixels and the centre, using the linearization base or the first order Taylor series.
In this paper an extended approximated non-linear polynomial model (second order Taylor series) for compressing images utilized. The rest of the paper organized as follows, section 2 contains comprehensive clarification of the proposed system; the results of the proposed system, is given in section 3.
2. The Proposed System

The main taken concerns in the proposed system are:

- It shows the effectiveness of the extended the approximated polynomial model of nonlinear based of second order Taylor series form to compress images compared to the approximated polynomial model of linear based.
- It describes a fully compression system that takes advantage of spatial redundancy (i.e., correlation) that modelled using six coefficients (a0,a1,a2,a3,a4,a5) compared to the linear model of three coefficients (a0,a1,a2)base. Along with eliminating unnecessary and unnoticeable redundancy (i.e., coding & psychovisual).

The steps below illustrate clearly the implementation of proposed system in details, the system layout shown in Figure (1):

**Step 1:** Load the original uncompressed gray image \( G \) of BMP format of size \( N \times N \).

**Step 2:** Partition the image \( G \) into nonoverlapping square fixed block of size \( n \times n \), then compute the estimated coefficients of the extended nonlinear model according to equations below, by first finding the \( a_1 \), \( a_2 \) and \( a_5 \) coefficients, such as:

\[
\begin{align*}
a_1 &= \frac{1}{n^2} \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} G(x, y)(x - xc)^2 \quad \text{.........(1)} \\
a_2 &= \frac{1}{n^2} \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} G(x, y)(y - yc)^2 \quad \text{.........(2)} \\
a_5 &= \frac{1}{n^2} \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} G(x, y)(x - xc)(y - yc) \quad \text{.........(3)}
\end{align*}
\]

where \( xc = \frac{n-1}{2} \) (.................(4))

For the coefficients \( a_0,a_3 \) and \( a_6 \), the extended approximation polynomial mode can be summarized as:

\[
\begin{align*}
V_1 &= a_0W_1 + a_3W_2 + a_4W_2 
V_2 &= a_0W_2 + a_3W_3 + a_4W_3 
V_3 &= a_0W_2 + a_3W_4 + a_4W_3
\end{align*}
\]

\[
\begin{align*}
W_1 &= n \times n \quad \text{........(5)} \quad \text{and} \\
W_3 &= \sum_{x=0}^{n-1} (x - xc)^4 + \sum_{y=0}^{n-1} (y - yc)^4 \quad \text{........(6)} \quad \text{Where} \\
V_1 &= \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} G(x, y) \quad \text{........(12)}
V_2 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (x - xc)^2G(x, y) \quad \text{........(13)}
V_3 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (y - yc)^2G(x, y) \quad \text{........(14)}
\end{align*}
\]

Where

\[
\begin{align*}
V_1 &= \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} G(x, y) \quad \text{........(12)}
V_2 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (x - xc)^2G(x, y) \quad \text{........(13)}
V_3 = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} (y - yc)^2G(x, y) \quad \text{........(14)}
\end{align*}
\]

In order to find the required coefficients, apply the Crammers rule, where:

\[
\begin{align*}
a_0 &= \begin{vmatrix} V_1 & W_1 & W_2 \\ W_1 & W_2 & W_3 \end{vmatrix} \quad \text{........(15)} \\
a_3 &= \begin{vmatrix} V_1 & W_2 & W_3 \\ W_2 & W_3 & W_4 \end{vmatrix} \quad \text{........(16)} \quad \text{and} \\
a_5 &= \begin{vmatrix} V_1 & W_2 & V_3 \\ W_1 & W_2 & W_3 \end{vmatrix} \quad \text{........(17)}
\end{align*}
\]
Here in the nonlinear form, the polynomial representation approximation model required 6 coefficients to represent each block compared to the linear model that require 3 coefficients. Basically, the nonlinear based needs times two of linear model coefficients, in spite of that, the compression performance not affected by these additional coefficients since the residual decrease due to increase the modelling efficiency. The main reason of estimating the coefficients decorrelation or removing of interpixel (spatial) redundancy (interpixel) is possible, by using the modelling concept.

Step 3: Quantize the estimated coefficients from step above using the popular scalar uniform quantizer, simply by dividing the each of the computed coefficients by the quantization step to eliminate the psychovisual redundancy. Here 3 quantization level of values required according to coefficients importance, one for \( a_0 \) values, other one for the \( a_1 \) & \( a_2 \) and the last one for \( a_3, a_4 \) & \( a_5 \) values. The quantizer/dequantizer as shown in equations (18-23).

\[
\begin{align*}
q_0Q &= \text{round}(\frac{a_0}{QS_{a0}}) \rightarrow a_0D = q_0Q \times QS_{a0} \quad \text{.........(18)} \\
q_1Q &= \text{round}(\frac{a_1}{QS_{a1}}) \rightarrow a_1D = q_1Q \times QS_{a1} \quad \text{.........(19)} \\
q_2Q &= \text{round}(\frac{a_2}{QS_{a2}}) \rightarrow a_2D = q_2Q \times QS_{a2} \quad \text{.........(20)} \\
q_3Q &= \text{round}(\frac{a_3}{QS_{a3}}) \rightarrow a_3D = q_3Q \times QS_{a3} \quad \text{.........(21)} \\
q_4Q &= \text{round}(\frac{a_4}{QS_{a4}}) \rightarrow a_4D = q_4Q \times QS_{a4} \quad \text{.........(22)} \\
q_5Q &= \text{round}(\frac{a_5}{QS_{a5}}) \rightarrow a_5D = q_5Q \times QS_{a5} \quad \text{.........(23)}
\end{align*}
\]

Where \( QS_{a0} \) quantization step of \( a_0 \) coefficient, \( QS_{a1} \) quantization step of \( a_1 \) & \( a_2 \) coefficients and \( QS_{a2} \) quantization step of \( a_3, a_4 \) & \( a_5 \) coefficients.

Step 4: Create the predicted image \( \tilde{G} \) as a nonlinear combination polynomial model of dequantized coefficients and pixel distance

\[
\tilde{G} = a_0W_1 + a_1(x-xc) + a_2(y-yc) + a_3(x-xc)^2 + a_4(y-yc)^2 + a_5(x-xc)(y-yc) \quad \text{.........(24)}
\]

The predicted image \( \tilde{G} \) corresponds to the modelled image representation that similar to the original image one, which is a weighted sum of coefficients and the pixel distance.

Step 5: Find the residual (prediction error) as a difference between original uncompressed image \( G \) and the predicted image \( \tilde{G} \)

\[
\text{Re } s = G - \tilde{G} \quad \text{.........(25)}
\]

The residual represents the information lost which can not be predicted accurately since the image features or characteristics cannot usually be fully described by a model where the details vary from part to part. The residual image acts as a quality indicator or measure of ”fit for the model, where a smaller residual indicates high compression gain, due to the adequate prediction model, while a larger residual indicates a low compression gain due to a poor prediction model [9].

Step 6: Quantize the residual image as in step 3 using the simple scalar uniform quantizer/dequantizer, such as:

\[
\text{Re } sQ = \text{round}(\frac{\text{Re } s}{QS_{Re}s}) \rightarrow \text{Re } sQD = \text{Re } sQ \times QS_{Re}s \quad \text{.........(26)}
\]

Where \( QS_{Re}s \) quantization step of residual image.

Step 7: Encode the lossily quantized information of estimated coefficients and residual image using the LZW coding to eliminate/remove the coding redundancy by converting them into variable bit length coding.

The decoder reconstructs the compressed image \( \hat{G} \) using only the information decoded of coefficients and residual, where the dequantized coefficients utilized to create the predicted image (see eq. 24), and then added to the dequantized residual image, such that:

\[
\hat{G} = \tilde{G} + \text{Re } sQD \quad \text{.........(27)}
\]
3. Experimental & Results

In general, as a lossy compression system utilized, the performance measure based on the objective fidelity criteria of Peak Signal to Noise Ratio (PSNR) (see equation 28), along with the Compression Ratio (CR) (see equation 29). Here various standard images were selected that characterized by variation in details, where the complex selected image of highly details such as Baboon (see figure 2a), the less complex one of medium details such as Lena (see figure 2b), and the simple image of small details such as Woman (see figure 2c), all the images are square gray scale images of size 256x256 pixels of 8 bit/per pixel.

\[
PSNR = 10 \log \left[ \frac{(255)^2}{\sum_{x=0}^{N-1} \sum_{y=0}^{N-1} [I(x,y) - \hat{I}(x,y)]^2} \right]
\] 

(28)

\[
\text{CompressionRatio} = \frac{\text{Size of Original Image}}{\text{Size of Compressed Information}}
\]

(29)

Table (1) illustrated the comparison performance of both linear and nonlinear polynomial coding techniques on the tested images, using two blocks of sizes 4×4 and 8×8, with quantization levels of coefficients of linear base equals to 1,3,3 and nonlinear base equals to 1,3,3,5,5,5, and with different quantization levels of residual image.

The results clearly showed the higher performance of nonlinear model base in terms of compression ratio and the quality that due to involving more coefficients of nonlinear terms where more accurately estimated the predicted image which leads to less residual error compared to the linear base model. Also the techniques of linear and nonlinear base implicitly affected by the block size and the quantization levels of the residual image, where for bigger block sizes and/or higher quantization levels higher compression achieved with lower PSNR quality image, and vice versa. Lastly, the results vary according to image details, where for the simple images more compression achieved compared to highly detailed image of low compression, since the techniques adopted of spatial domain base that directly affected by the image contents. Figure (3 a,b,c) showed the performance comparison between the linear and nonlinear base of the three tested images using block size of 4×4.
Fig(2): Tested images

Fig(3): Comparison performance between linear & nonlinear polynomial techniques of 4x4 block.
Table (1): Comparison performance between linear and non-linear polynomial coding techniques, using two blocks of sizes 4×4 and 8×8, with quantization levels of coefficients of linear base equals to 1,3,3 and nonlinear base equals to 1,3,3,5,5,5, and with different quantization levels of residual image.

<table>
<thead>
<tr>
<th>Test Images</th>
<th>Quantization Residual</th>
<th>Non Linear Polynomial Coding</th>
<th>Linear Polynomial Coding</th>
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<tr>
<td></td>
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<td>Block Size 4×4</td>
<td>Block Size 8×8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Quantization Coefficients 1,3,3,5,5,5</td>
<td>Quantization Coefficients 1,3,3</td>
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<td>PNR</td>
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