Available Online at www.ijcsmc.com

International Journal of Computer Science and Mobile Computing

A Monthly Journal of Computer Science and Information Technology

ISSN 2320–088X

IMPACT FACTOR: 5.258

IJCSMC, Vol. 5, Issue. 3, March 2016, pg.197 – 201

The Minority Game Revisited

Syed Md. Galib[1], Md. Shafiul Alam Forhad[2], Md. Bellal Hossain[3], M. M. Rahaman[4]

[1,2]Department of Computer Science and Information Technology, Patuakhali Science and Technology University, Bangladesh

[3]Department of Mathematics, Patuakhali Science and Technology University, Bangladesh

[1] galib@pstu.ac.bd; [2] forhad090463@gmail.com; [3] bellal77pstu@gmail.com; masudurpstu@gmail.com [4]

Abstract— Since its introduction, the minority game (MG) has been receiving much attention by researchers of different fields of research. Therefore, in this paper, we reviewed MG with its different variations so that the working mechanism of this evolutionary game becomes clear. For that, we first explained how the concept of bounded rationality comes when Arthur proposed this El-Farol Bar problem and how this model can be simplified using MG.

Keywords— Game Theory, Evolutionary Game Theory, El-Farol Bar Problem, Minority Game, Multi-agent System

I. INTRODUCTION

During a decision making scenario, a decision maker may take decision based on the information she possesses. If her decision affects other individuals' decisions, we call this situation as a game. Game theory (GT) concerns itself with the situations or games where there are some players and their decisions may affect the decisions of each other. Each player has her own interest i.e. players are self-interested. They want to maximise their own utility and they usually do not care about other players' utilities. The players usually do not communicate with each other. In such a non-cooperative scenario where players do not communicate, the players must make their own decisions independently. The players of a game may have a set of possible strategies to choose from in an attempt to maximise their utilities which are jointly dependent on the decisions of all players involved in the game[1].

From the above discussion follows that a game consists of three attributes[2].

- Players - the set of participants in the game.
- Strategies - a set of options how each player acts or takes decisions in the game. Strategies can be either pure strategies or mixed strategies. Pure strategies are strategies which players choose deterministically. Mixed strategies are randomisations over pure strategies i.e. in this case, players assign a probability on the pure strategies to choose from.
- Utility - the payoff for using a strategy after all players' actions.

Thus we can define a game Γ as

$$\Gamma = (N, (S_i, u_i)_{i\in N}) \tag{1}$$

where,

- N is the (finite) set of players involved in the game
- S_i is the (finite) set of pure strategies for player i
- u_i is the utility for using S_i by player i

Evolutionary game theory (EGT) deals with evolutionary nature of games where a large number of players make their decisions repeatedly. The Minority Game (MG) [3] is an game which was proposed by Challet and

Zhang in 1997. Since its introduction mainly in the physics field of research, it has been applied to various research areas such as – economics [4, 5], statistical mechanics[6] and road traffic optimisation[7-9]. Therefore, it has wide acceptance to the researcher communities of different fields of research and also has potential to be applied in other areas. However, Challet and Zhang [3] proposed MG as a simplification of Arthur's El-Farol Bar problem[10].

II. EL-FAROL BAR PROBLEM

Arthur[10] argued that in complicated or ill-defined situations, human beings do not reason deductively but inductively. Deductive reasoning is the process of making a decision based on some generalised facts whereas inductive reasoning makes a conclusion based on experiential or historical facts and thus is probabilistic in nature.

Arthur was inspired by the El-Farol Bar in Santa Fe where every Thursday evening Irish music was played. People went to the bar to enjoy the evening with the music. Arthur formulated the El-Farol Bar Problem as an inductive reasoning and bounded rationality problem[10]. The El-Farol bar has limited capacity and if the number of patrons exceeded the capacity, the bar became crowded and the patrons did not enjoy themselves. Arthur defined the problem as follows. A number of players or agents decide independently and without any communication, each week, whether to go to the bar. The agents predict the number of attendants in the bar and if the prediction concludes that there will be less people than the capacity allows, he will go to the bar, otherwise he will stay at home. Hence, there are two possible actions for each agent: `go to the bar' or `stay at home'.

Arthur pointed out two interesting features of this problem[10]. Firstly, it is not possible for the agents to reason deductively as an agent cannot predict exactly what the other agents are going to do. As there is no correct expectation, the problem is ill-defined and complex. In this kind of situations, agents attempt to simplify the problem by using their own beliefs or histories. The agents cannot predict other agents' action and they do not have enough information to come to process a proper deduction. In such a situation, the agents use inductive reasoning.

The second interesting feature is a forced differing of expectations. If all patrons expect that no one will go to the bar, all will attend which would invalidate their expectations or beliefs. Similarly, if all patrons expect that all will go to the bar, no one will attend - which again will invalidate their beliefs. Thus the expectations are forced to be different.

Arthur observed that the agents can have a bag of ideas or a set of predictors to predict the number of attendants in the bar. Based on the prediction, a particular agent decides if she would go to the bar. If the predicted attendance is smaller than the capacity of the bar, the agent will go to the bar. The agent stays at home if the prediction exceeds the capacity. The agents keep track of the performance of their sets of predictors and decide to go or stay according to its most accurate or active predictor. After all decisions have been made, the agents come to know about the current attendance and update the fitness or accuracy of their selected predictors.

The predictors can be of many types. Table I shows some types of predictors with the predictions and the decision by an agent for that prediction for $N = 100$ agents and a capacity of the bar of 60. Let us assume the following history of attendance.

……..31 27 28 16 36 90 71 30 33 80 75 49 31 47 79 38 83 21

The agents take their decisions based on the history of the attendance in the bar. which is mapped to a prediction by their own predictors.

TABLE I
SOME EXAMPLE TYPES OF PREDICTORS FOR EL-FAROL BAR PROBLEM. THE TOTAL NUMBER OF AGENTS IS 100, THE BAR CAPACITY IS 60

Predictor	Prediction	Decision of the agent
The average of last 3 weeks' attendance	47	go to the bar
The same as 2 weeks ago	83	stay at home
The average of last 5 weeks' attendance	54	go to the bar
Mirror of last weeks' attendance around 50	79	stay at home
A random number	61	stay at home
The same as 3 weeks' ago	38	go to the bar
Use linear trend of last 5 weeks' attendance	40	go to the bar

Arthur's experimentation showed that the bar attendance fluctuates around 60[10]. Arthur concluded that the predictors self-organise into an equilibrium and the mean attendance converged to 60 which is the bar's capacity. However, the memory requirement for Arthur's model is huge. If there are N agents who memorises last m weeks' attendance and based on that they want to predict a number between 0 and N, the total number of possible combination is N^{N^m}. For this reason, Challet and Zhang simplified this model using a binary evolutionary game – the minority game[3].

III. MINORITY GAME

MG is a game theoretic model introduced by Challet and Zhang [3], to simplify Arthur's El-Farol Bar Problem[10]. In MG N (which is an odd number) agents play the game (Fig. 1) repeatedly. At each time, each agent takes an action represented by either +1 or -1 for going to the bar or staying at home. The agents on the minority side win while the others lose.

In the simplest version, all winners gain a point. Similar to any competitive game, the objectives of the agents are to gain as many points as possible. Thus, from a player's individual point of view, if he can score more points, he is the only one to benefit. However, from the social perspective, if almost half the population of agents can score points, they all benefit.

The players make their decisions based on the common knowledge of past records which is `+1' or `-1' denoting the winning side. If the agents taking decisions +1 were in the minority last time, the history will be +1, not the number of agents taking the action +1 as would have been in the case of Arthur's formulation. The history being the winning alternative simplifies EFBP where agents received the feedback about the exact number of attendants in the bar.

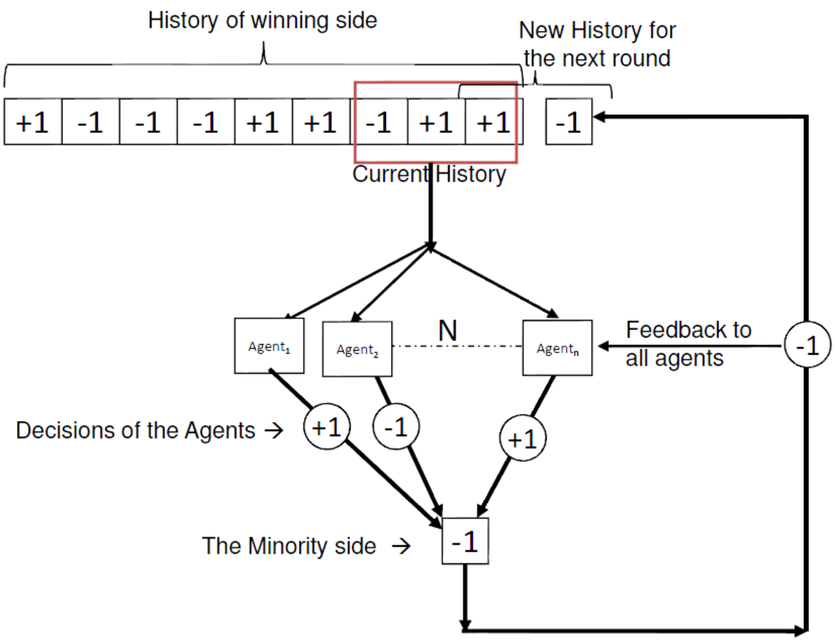

Fig. 1: A setup of the Minority Game

As in Challet and Zhang's model, the alternative chosen by the minority of agents is the winning alternative and no capacity constraint is imposed on the venue. The history of previous winners is denoted as a binary sequence with +1 and -1. The agents are provided with the common history of the last *m* winning sides. The history is the m-bit string of +1s and -1s. Each agent has a finite number of strategies which map the action +1 or -1 for the next time step based on m-bit history. Thus predictor or strategy length becomes 2^m which is significantly smaller than N^m in the El-Farol Bar Problem. Table II shows an example of three strategies for $m = 3$.

The left hand side of Table II shows all possible combinations of the history for $m = 3$ and the right hand side is the action predicted for that particular combination of the history. In the traditional minority game, the strategies are initialised randomly and the agents cannot change their strategies. There is a chance of having the same strategy for different agents; however, for a moderately large number of total strategies, this is less probable.

TABLE III
AN EXAMPLE OF PREDICTOR FOR m = 3

History				Predictor 1	Predictor 2	Predictor 3
-1	-1	-1		1	1	-1
-1	-1	1		1	-1	-1
-1	1	-1		-1	-1	-1
-1	1	1		-1	1	-1

1	-1	-1		1	1	-1
1	-1	1		-1	1	1
1	1	-1		1	1	1
1	1	1		1	-1	1

Two extreme cases exist in Minority Game (MG). In the first case, one single player makes one of the two choices whereas all other players choose the other option. This player will then be the only one to score. Another extreme case is when there are (N-1)/2 players in one side and (N+1)/2 players at the other side and almost half of the agents score. The first case is positive from the individual's point of view, while the second case is beneficial from a social point of view. Experiments show that agents playing MG distribute themselves almost evenly over the two groups and achieve good average gains. Challet and Zhang[3] also showed that constantly choosing one alternative will not benefit an agent who may think that he can win at least half the time. If some agents choose one alternative every time and happen to win by becoming the minority, other agents will conclude that they are losing more often by choosing the other alternative. Thus, these agents will choose the opposite alternative and the agents who were in the minority would be in the majority side and in consequence would lose. Therefore, Challet and Zhang commented that any obvious advantage would be '*arbitraged away*' and the game is symmetrical for both alternatives.

Challet and Zhang[3] analysed the size of the memory to determine how long the history should ideally be. How many past iterations should the agents remember? The authors experimented using different memory sizes. Their results indicated that for any memory size, the agents distribute themselves. However, fluctuation is present. The game is more efficient when there is small fluctuation as a larger number of agents have won. On the other hand, large fluctuation indicates that even more agents could have won by being on the minority side. Challet and Zhang's[3] experiments indicate that a smaller memory size introduces larger fluctuation. Even though the agents are selfish in nature in the sense that they do not consider other agents' gains or advantages, they self-organise over time and distribute themselves almost evenly between the two alternatives. This indicates that using the MG technique may be helpful to achieve a good distribution of cars on the roads where each traveller chooses her route independently and selfishly.

How many strategies or predictors each agent should have was also analysed by Challet and Zhang[3]. They experimented with several settings where in each setting all agents have a different number of strategies. The results indicated that in the scenario where each agent has two strategies, the average gain of the agents are the highest. Therefore, the authors concluded that a set of two strategies per agent is sufficient to play Minority Game.

Challet and Zhang's[3] traditional MG has homogeneous agents with the same number of predictors and the same memory sizes or same history length for all agents. In a mixed population, where agents may have different memory sizes, agents with bigger memory i.e. a longer history win more often[11]. Through experiments, Araujo and Lamb[11] showed that if all other agents have a memory size less than or equal to three, the agents who have the bigger memory size win more often.

Cavagna[12] pointed out an interesting aspect regarding the memory or the history in MG. The author simulated different experiments with different numbers of agents to show that even if the agents were provided with fake history, the results were the same as the results of traditional MG of Challet and Zhang. This is true as long as all the agents receive the same fake history[12].

In traditional MG[3], agents initially select S strategies randomly from the pool of strategies. An agent uses the best strategy from this set. The best strategy is selected based on the scores of the strategies. After each iteration when all agents make the decisions, each agent evaluates her strategies regardless of whether she has used it. The strategies receive scores based on their predictions if they would have been used. The strategy with the highest score is chosen by the agent as the best strategy to be used in the next iteration. The set of strategies remains the same throughout the whole period of the game. Thus if it happens that an agent picked strategies which constantly predict the losing alternative for some histories, the agent wins less often. Araujo and Lamb[13] analysed an evolutionary version of MG which demonstrated that if the agents can change their predictors after every L rounds, the agents perform better than when they apply the traditional MG, if L is small enough.

Sivanadyan and Sethares[14] introduced a probability-based approach which divides the agents into two groups - after some iterations - where one group always chooses one side and the other chooses the opposite. The authors modified the decision making of the agents. Instead of using predictors, the agents choose an alternative with a probability. The agents update the probability of choosing the alternative after each iteration. The authors showed that the agents divide into two equally sized groups after some iterations - each group chooses one alternative. The minority side is decided by only one agent who switches between the two groups and in consequence, that agent is never on the minority side. The probability-based approach takes less time to converge to an equilibrium than the traditional MG. However, in road traffic scenario, it is not desirable for a small group of travellers to be on a crowded route at all times.

Chow and Chau[15] proposed multiple choice minority game (MCMG) as an extension of MG. In MG, there are only 2 alternatives to choose from whereas in MCMG each agent has a choice between several alternatives. The authors described MCMG as a game where N players have to choose from N_c rooms/choices independently and repeatedly. The authors assumed that N_c is a prime number. The players in the room chosen by the minimum number of players are the minority and win at that iteration. The winning room is provided to the players as the history. The players map the past m iterations' history to choose for the current round using S strategies/predictors. The strategies consist of weights which are chosen uniformly randomly. The choice by a player using a given strategy is the weighted sum of last m iterations' winning side added with the bias. While calculating the weighted sum of the last m iterations' minority side, the weights are defined by the strategies used. Chow and Chau[15] compared their results of MCMG for two rooms with MG by Challet and Zhang[3] and showed that the mean attendance for two room MCMG fluctuated around the expected value of *N/2* resembling the mean attendance for MG. Similarly, the variance of the attendance of two room MCMG showed matching behaviours with MG.

IV. CONCLUSIONS

Game theory is the study of *games* where *players* take decisions based on their strategies which usually maximize their utilities. The minority game (MG) is a non-cooperative game where players or agents decide one of the two available alternatives so that they may fall in the minority group. The minority group wins and gains utility. Though simple in nature and introduced in the research area of physics, the MG has received much attention by researchers of different fields since its introduction. The MG has been applied to model market mechanisms[4, 5] , to statistical mechanics[6] and to optimise road traffic through traffic assignment[7-9]. Although it has been modified in some cases, MG has wide acceptance to the researchers of different fields. For this reason we reviewed MG and its different modifications in this paper so that the readers can have an insight into how the model works.

REFERENCES

[1] T. Schlegel, "Decentralised Agent-based Resource Allocation in Open and Dynamic Environments," PhD Thesis, Faculty of Computer Science and Information Technology, Swinburne University of Technology, Melbourne, Australia, 2010.

[2] D. Easley and J. Kleinberg, *{Networks, Crowds, and Markets: Reasoning About a Highly Connected World}*: Cambridge University Press, 2010.

[3] D. Challet and Y. C. Zhang, "Emergence of cooperation and organization in an evolutionary game," *Physica A,* vol. 246(3-4), p. 12, 1997.

[4] D. Challet, M. Marsili, and Y. C. Zhang, "Modeling market mechanism with minority game," *Physica A,* vol. 276, pp. 284-315, 2000.

[5] D. Challet, M. Marsili, and Y. C. Zhang, *Minority Games: Interacting agents in financial markets*: Oxford University Press, 2005.

[6] E. Moro, "The Minority Game: an introductory Guide," *Advances in Condensed Matter and Statistical Physics,* 2004.

[7] S. M. Galib and I. Moser, "Road traffic optimisation using an evolutionary game," presented at the Proceedings of the 13th annual conference companion on Genetic and evolutionary computation, Dublin, Ireland, 2011.

[8] A. L. C. Bazzan, R. H. Bordini, G. K. Andrioti, R. M. Vicari, and J. Wahle, "Wayward agents in a commuting scenario (personalities in the minority game)," in *MultiAgent Systems, 2000. Proceedings. Fourth International Conference on*, 2000, pp. 55-62.

[9] R. Kitamura and S. Nakayama, "Can travel time information influence network flow? - Implications of the minority game," *Transportation Research Record,* pp. 14-20, 2007.

[10] W. B. Arthur, "Inductive Reasoning and Bounded Rationality," *American Economic Review (Papers and Proceedings),* vol. 84, p. 6, 1994.

[11] R. M. Araujo and L. C. Lamb, "On the Use of Memory and Resources in Minority Games," *Acm Transactions on Autonomous and Adaptive Systems,* vol. 4, pp. -, May 2009.

[12] A. Cavagna, "Irrelevance of memory in the minority game," *Physical Review E,* vol. 59, pp. R3783-R3786, 04/01/ 1999.

[13] R. M. Araujo and L. C. Lamb, "Towards Understanding the Role of Learning Models in the Dynamics of the Minority Game," *Proceedings of the 16th IEEE International Conference on Tools with Artificial Intelligence,* p. 10, 2004.

[14] T. Sivanadyan, "A Decentralized Optimal Solution to the Minority Game," Master's Thesis, ECE Department, UW-Madison, 2004.

[15] F. K. Chow and H. F. Chau, "Multiple choice minority game," *Physica A: Statistical Mechanics and its Applications,* vol. 319, pp. 601-615, 2003.